Joint Wideband Spectrum Sensing in Frequency Overlapping Cognitive Radio Networks Using Distributed Compressive Sensing

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***Abstract*—The emerging paradigm of open spectrum market calls for quick, efficient and dynamic approach for spectrum sensing. Conventional spectrum sensing methods for cognitive radios are capitalized over the narrow band sensing without addressing the wideband spectrum sensing. In wideband networks, one by one scanning of spectrum is unattractive because of its complexity, agility constraints, and data acquisition cost. Existing wideband spectrum sensing schemes do not exploit the gain of joint sparsity in the frequency overlapping networks. In this paper, wideband spectrum sensing for frequency overlapping cognitive radio networks using the emerging compressive sensing paradigm and joint reconstruction is proposed. Simulation results verify the effectiveness of proposed joint spectrum sensing approach in jointly sparse frequency overlapping cognitive radio networks.**

I. Introduction

The increasing demand for wireless resources and spectrum have created spectrum scarcity. However, spectrum utilization studies show that these scarce wireless spectrum has been distributed and used inefficiently [1], [2]. This bottleneck in spectrum scarcity and inefficient usage is addressed by the dynamic spectrum access policy. Cognitive radio (CR) with ability to sense unused spectrum and opportunistically transmit over spectrum holes is proposed in [3], [4]. The fundamental challenge in the cognitive radio implementation is detection of the vacant spectrum (holes) [5]. The current trends in spectrum sensing and cognitive radios are well explained in many survey reports [6], [7]. Many of the spectrum sensing algorithms deal with the narrow band sensing which are tailored energy detector and power spectral density of the narrow band signal. Wideband spectrum sensing requires fast and dynamic spectrum analysis over larger spectrum band. Recent paradigm in sparse sampling, compressive sensing (CS) [8], [9] provides solution to sparse signal reconstruction as an optimization problem. In [10], [11] CS sampling, forward differentiation and singular value decomposition methods are used for wideband spectrum sensing purpose. These approaches provide wideband spectrum sensing in a simple individual network. To eliminate the need of high speed analog to digital converters and digital signal processors in wideband sensing, techniques such as random demodulators, parallel signal processing are proposed in [12], [13].

The NTIA's frequency allocation chart [14] shows many networks overlap in the frequency zone because of spectrum scarcity. The spectrum overlapping or jointly sparse frequency overlapping network in cognitive radio network comes into picture because of spatial diversity of primary and secondary transmission power [15]. In this paper, we propose joint reconstruction for wideband spectrum sensing in frequency overlapping networks using distributed compressive sensing.

The rest of the paper is structured as follows. Section II provides mathematical formulation of the general spectrum sensing problem in a single network. In Section III, we introduce the frequency overlapping network and formulate the spectrum sensing problem for it. Individual reconstruction scheme and joint reconstruction for the compressed measurement are presented and compared. Section IV provides simulation results for the proposed joint wideband spectrum sensing in frequency overlapping networks and finally, Section V concludes the paper.

II. Problem Statement

Let us consider a wideband communication model with primary and secondary (cognitive radios) users coexistence as shown in the Fig. 1. The total communication bandwidth of the system is divided into N subbands each centered at frequency f_n, where $n = 1, 2, 3, \ldots, N$. Very few of the N subbands are occupied by the primary users at a given geographical and temporal region. Let us suppose, out of N subbands, $S << N$ are occupied by primary users during sensing time. The unoccupied channels by primary users over given spatia-temporal region called, *spectrum holes*, are opportunistically accessed by the cognitive users keeping the rights of the primary safe. The cognitive radios in the system need to detect these spectrum holes for secondary communication.

At time t, the received signal at m^{th} cognitive user can be expressed as:

$$y_m(t) = \sum_{n=1}^{N} x_n(t) * g_{nm}(t) + w_m(t), \quad (1)$$

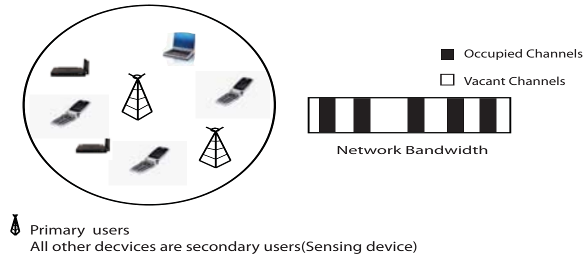

Fig. 1. Primary and secondary users coexistence

where, $*$ represents the convolution, $x_n(t)$ is the signal of n^{th} primary user, $g_{nm}(t)$ denotes the channel gain response, and $w_m(t)$ is additive white gaussian noise with zero mean and variance of σ_w^2. In frequency domain, (1) can be represented as:

$$y_f^{(m)} = \sum_{n=1}^{N} \mathbf{D}_g^{(nm)} x_f^{(n)} + w_f^m, \tag{2}$$

where $\mathbf{D}_g^{(nm)}$ is a diagonal $N \times N$ channel gain matrix between n^{th} primary and m^{th} CR. x_f and w_f represent corresponding frequency response of $x(t)$ and $w(t)$, respectively and elements $g_f^{(nm)}$ of $\mathbf{D}_g^{(nm)}$ are given by:

$$g_{(i,j)} = 0; \qquad i \neq j;\ and\ i,j \in \{1,2,\dots N\}, \tag{3}$$

$$and, \quad g_{(i,j)} = g_{f(i,j)}; \qquad i = j;\ and\ i,j \in \{1,2,\dots N\}. \tag{4}$$

However, we know that at any time t, only few of the N channels are occupied . Let $\hat{S}$ be the set of occupied channels such that $\hat{S} \subset \hat{N}$. $\hat{N}$ is the set of bands under consideration. Thus for all $n : n \notin \hat{S}$

$$x_n(t) = 0. \tag{5}$$

From (5), $\hat{N}$ is sparse. Accordingly, (1) and (2) reduce to,

$$y_m(t) = \sum_{s \in \hat{S}} x_s(t) * g_{sm}(t) + w_m(t), \tag{6}$$

$$and, \quad y_f^{(m)} = \sum_{s \in \hat{S}} \mathbf{D}_g^{(sm)} x_f^{(s)} + w_f^m. \tag{7}$$

This sparseness of signal in the frequency domain makes CS possible for the spectrum sensing purposes in cognitive radio network. The frequency response of the channels occupied by the primary users are non-zero values, whereas, those of vacant channels are zero. Hence, the total frequency response of the signal under consideration is a sparse signal. In CS, instead of taking point by point samples as in conventional sampling, each sample taken is linear functional of the sparse signal. Consider the problem of reconstructing $N \times 1$ length, S sparse signal X. Let us consider an $M \times N$ dimension, where $M < N$, sensing matrix Φ. We can obtain M compressed measurements, Y, using, $Y = \Phi X$. Since, $M < N$, the recovery of X from compressed measurements Y is ill-posed in general. Interestingly, provided X is sparse in some domain and the measurement matrix Φ satisfies the restricted isometry property (RIP) [8], the signal X can be recovered from the measurement vector Y. The recovery of S non-zero elements of signal X is actually the solution of the l_0 norm minimization problem. Unfortunately, solving l_0 is prohibitively computationally complex. However, the approximate solution of X can be obtained using l_1 minimization as:

$$\hat{X} = arg\ min||X||_{l_1}, \quad s.t. \quad Y = \Phi X, \tag{8}$$

For secondary communication in the cognitive radio network, finding the set $\hat{S}$ is the most important and first requirement. The complexity of spectrum sensing depends upon the requirements of an application. In cognitive radio spectrum sensing, our primary concern is finding which of the S bands among N are occupied rather than the exact signal strength of the the occupied channels. In cognitive radio network, the problem of spectrum sensing using energy detector, at each m^{th} CR boils down to distinguishing between binomial hypotheses. The signal energy of each of the subband is compared with the threshold λ_e which is the function of noise and channel characteristics. If the received signal is greater than λ_e the channel is said to be occupied else it is taken as vacant. Finding an optimal λ_e is an agenda in the communication channel modeling research [16].

In conventional spectrum sensing, each secondary user senses and detects each of the band individually. This requires large number of measurements in the system and increases data acquisition cost [7], [10]. Besides, each sensing device/cognitive radio[1] needs to have sensing bandwidth entirely over the communication band making the sensing process more prone to noise. Moreover, all the CRs have to be silent and synchronized during the sensing period. In large overlapping networks, or in spatially distant CRs, synchronization cannot be guaranteed due to spatial diversity of primary transmission power. This creates jointly sparse frequency overlapping networks over large spatial domain [15].

III. Proposed Wideband Compressive Spectrum Sensing in Frequency Overlapping Networks

In this paper, we present a novel, joint wideband spectrum sensing scheme for frequency overlapping cognitive radio network, based upon the new sparse signal acquisition scheme called compressed sensing for which signal reconstruction is an optimization problem. We extend our work in wideband compressive sensing for cognitive radios [17] into a frequency overlapping network and present joint reconstruction scheme for spectrum sensing in frequency overlapping networks.

A. Distributed Wideband Spectrum Sensing in Frequency Overlapping Network

The NTIA's frequency allocation chart clearly shows the frequency overlapping over different system protocols to meet

[1]The terms sensing device and cognitive radio have been used interchangeably in this paper

the band scarcity issue. Let us consider two network systems S_1 and S_2 with some overlapping operating bands as shown in Fig. 2. We call the networks S_1 and S_2 as the frequency overlapping networks and denote it with network H_N. The theoretical backgrounds on joint sparse signal can be found in literatures [18], [19]. Let B_{s1} and B_{s2} represent the spectrum band of S_1 and S_2 respectively, where $|B_{s1}| = N_1$ and $|B_{s2}| = N_2$. B_{sc} denotes the frequency overlapping between S_1 and S_2 and $|B_{sc}| = N_c$. The total number of bands (channels) under consideration is : $N_T = N_1 + N_2 - N_c$. In jointly sparse frequency overlapping networks, for each of the network, (7), takes the form of:

$$y_{if}^{(m)} = \sum_{s \in \hat{S}_i} \mathbf{D}_{gi}^{(sm)} (x_{if}^{(s)} + x_{cf}^{(s)}) + w_f^m, \tag{9}$$

where, $i = 1,\ 2$ refers to corresponding network, $x_{if}^{(s)}$ and $x_{cf}^{(s)}$, denote the spectral innovation of i^{th} network and joint sparse portion, respectively, as illustrated in the Fig. 2, and all other notations have same meaning as in (7). We consider, each of the network consists of M_s sensing devices and each sensing device takes m_s compressed measurements. So total number of measurements taken in each network $= M_s \times m_s = M$.

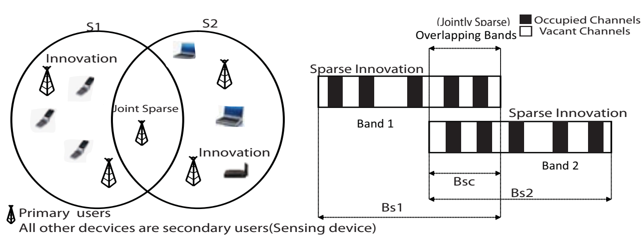

Fig. 2. Schematic of overlapping networks and overlapping spectrum bands

Let $X_1 = [x_i^{(1)}]_{N_1 \times 1}$ and $X_{(2)} = [x_i^{(2)}]_{N_2 \times 1}$ represent the test statistic for the spectrum sensing in two networks $S_1\ and\ S_2$ respectively and $X_{(c)} = [x_i^{(c)}]_{N_c \times 1}$ represents that of overlapping portion. A vector V of length N is said to be K sparse if V contains only K non-zero elements, i.e. $\|V\|_{l_0} = K$, where, l_0 denotes norm zero. Also, the support of a vector, $V = [v_i]_{N \times 1}$ is defined as : $supp(V) = \{i \quad, v_i \neq 0, i = 1, 2, \ldots N\}$. Let, $|supp(X_1)| = K_1\ and\ |supp(X_2)| = K_2$. K_1 and K_2 denote number of occupied channels in network 1 and 2, respectively. It should be noted that in spectrum sensing for cognitive radios, our objective is to find the $supp(X_1)\ and\ supp(X_2)$ and hence detect primary users and find the spectrum holes.

In compressive sensing, it is the method of data acquisition which makes it distinct from conventional sampling approaches. In the following subsections, we describe the sampling approach, the structure of sparse sampling matrix Φ, data acquisition techniques and decoding approaches.

1) Sensing Matrix Φ: In previous works [20], [8] the mathematical models of compressive sensing have been explained thoroughly. From the implementation point of view the physical realization of sampling matrix Φ is an important issue. The sensing matrix $\Phi_{M \times N}$ in our model is the element-wise combination of the two matrices: random frequency selective matrix $F_{M \times N}$ and channel response matrix $H_{M \times N}$. i.e

$$\Phi = F(.\star)H, \tag{10}$$

where, $(.\star)$ represent element wise product. We consider each of the M_s sensing device/CR consists of m_s filter banks, where $M_s \times m_s = M$. Each filter bank is collection of random bandpass filter tapped at L random bands. For simplicity, we assume the filters are ideal filters with unity gain and zero phase. Hence, F is a binary matrix with constant row weight L. Also, if L_m denotes the the set of band index of the filters in the m^{th} frequency selective filter bank, then:

$$F_{m,n} = 1 \quad ; if, n \in L_m, \tag{11}$$

$$else,\, F_{m,n} = 0. \tag{12}$$

Similarly, the channel response matrix is defined by:

$$H = h_{m,n}, \quad m = 1, 2, \ldots M \quad and \quad n = 1, 2, \ldots N, \tag{13}$$

where, $h_{m,n}$ is the channel response between m^{th} sensing device and the n^{th} primary signal, and is function of the channel modeling. From (10) and (11), we can have :

$$\Phi_{m,n} = F_{m,n} \times H_{m,n} \quad ; if F_{m,n} = 1, \tag{14}$$

$$else,\, \Phi_{m,n} = 0. \tag{15}$$

Hence, the sensing matrix Φ is a constant row weight matrix.

2) Compressed Measurement Y: Let the sensing matrix Φ for network systems S_1 and S_2 be represented by $[\Phi_1]_{M_1 \times N_1}$ and $[\Phi_2]_{M_2 \times N_2}$ respectively, with the characteristics as explained in Section III-A1. For ease in calculation, we assume $M = M_1 = M_2\ and\ N_1 = N_2 = N$. Each sensing device samples the spectrum bands in the corresponding network system in S_1 and S_2. Each sensing device gives m_s compressed measurements and each network consists of M_s sensing devices. For each system, the total number of compressed measurements sent to the individual controller unit is then $M = M_s \times m_s$. The i^{th} compressive measurement at m^{th} sensing device is given by:

$$y_m^{(i)} = \Phi_{(m,:)} \times X, \tag{16}$$

$i = 1, 2 \ldots m_s, m = 1 : number\ of\ sensing\ devices\ (M_s)$
Hence, Y_1 and Y_2, denoting the compressed measurement at network system 1 and 2 respectively, can be written as:

$$Y_i = \Phi_i \times X_i; \quad i = 1, 2. \tag{17}$$

Similarly, in case of the noisy measurements, it is affected with additive white gaussian noise of zero mean and variance σ^2, $W(0, \sigma^2)$.

$$Y_i = \Phi_i \times X_i + W_i; \quad i = 1, 2. \tag{18}$$

3) Compressive Sensing Decoding: The solution to the compressive sensing decoding is an optimization problem. CS decoding algorithm based upon the norm optimization like Basis pursuit (l_1) minimization is discussed in [9], [8], [21].

In the followings, we first provide a quick reference to individual compressive spectrum sensing and individual reconstruction, then we illustrate the joint reconstruction scheme for the frequency overlapping networks.

3.a. Individual Reconstruction

In individual reconstruction scheme, each network reconstructs its compressively sensed test statistics individually without cooperating with other networks and the decision about the spectrum occupancy is made accordingly using thresholding [16]. The reconstructed test vectors in individual reconstruction is give by:

$$\hat{X}_i = arg\ min \|X_i\|_{l_1} \quad s.t. \quad Y_i = \Phi_i X_i \quad ; i = 1, 2, \quad (19)$$

where as in case of the noisy measurements the optimization constraint is minimized as:

$$\|Y_i - \Phi_i X_i\|_2 \leq \sigma^2, \quad (20)$$

3.b. Joint Reconstruction

The number of required measurements for CS reconstruction is a function of the sparsity of the signal. It has been shown that the number of samples required for the CS reconstruction is in the order of $CKlog\left(\frac{N}{K}\right)$ [8], [19]. In overlapping networks, the individual reconstruction requires redundant numbers of samples for reconstruction. In individual reconstruction, the number of measurements required depends on $(K_1 + K_2)$. In [15], the LASSO algorithm with iterative user consensus is used to detect the overlapped bands. However, the advantage of common sparse elements in joint reconstruction is not exploited, and individual reconstruction is required in each network. In joint reconstruction, the number of measurements required for reconstruction depends on $(K_1 + K_2 - K_c = K_T)$. It has been shown that the the number of required measurements for CS reconstruction depends upon the sparsity, hence the joint reconstruction will have the measurement gain. Moreover, only one joint optimization is performed for the reconstruction of the both networks. We implement joint reconstruction scheme for spectrum sensing in frequency overlapping networks and compare it with the conventional individual reconstruction scheme and the iterative LASSO consensus algorithm [15]. In joint reconstruction scheme, cognitive users in each network take the compressed measurements of spectrum in their network. The CS measurements are sent to a common controller unit.

Let the measurements for the joint reconstruction be denoted by Y as,

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}, \quad (21)$$

where, Y_1 and Y_2 are compressed measurements of networks S_1 and S_2, respectively. Joint reconstruction matrix Φ_{joint} for reconstruction of the spectrum test statistics,

$$X := \begin{bmatrix} X_{i1} \\ X_c \\ X_{i2} \end{bmatrix},$$

be represented as:

$$\Phi_{joint} = \begin{bmatrix} \Phi_A & \Phi_{C_1} & \Phi_{null} \\ \Phi_{null} & \Phi_{C_2} & \Phi_B \end{bmatrix}. \quad (22)$$

If I_c denotes the set of the overlapping bands of two networks, $\hat{X}_{i1}$ and $\hat{X}_{i2}$ denote innovation bands of network 1 and 2 respectively, and $(\phi)_j$ denotes the j^{th} column of the Φ, then :

$$\begin{aligned} \Phi_A &= (\Phi_1)_j & j &\in \hat{X}_{i1}, \\ \Phi_B &= (\Phi_2)_j & j &\in \hat{X}_{i2}, \\ \Phi_{C_1} &= (\Phi_1)_j & j &\in I_c, \\ \Phi_{C_2} &= (\Phi_2)_j & j &\in I_c, \end{aligned} \quad (23)$$

and Φ_{null} are null matrices. Then the joint reconstruction optimization for X is performed as:

$$\hat{X} = arg\ min \|X\|_{l_1} \quad s.t. \quad Y = \Phi_{joint} \times X. \quad (24)$$

In case of noisy measurements the constraint of optimization is modified accordingly as in (20).

IV. Simulation and Results

For evaluating our performance we define following performance measurement parameters.

Sampling Rate ($S.R = \frac{2M}{N_T}$): Sampling rate is defined as the ratio of the number of compressed measurements to the total number of channels.

Probability Of Detection (POD): It is the ratio of total number of hits to the sums of total hits and miss. Hit is an event when we decide the presence or absence of primary user correctly, whereas, any other wrong decision is termed as miss event.

Error of Reconstruction (EOR): EOR is the ratio of energy difference between reconstructed and original signal to the energy of the original signal.

Sparse Overlapping Factor ($SOF = \frac{K_c}{K_T}$): SOF is the ratio of number of occupied channels in the overlapping bands to the total number of occupied channels in the network.

Measurement Gain (MG): For the given probability of detection, measurement gain is defined as: $MG = 1 - \frac{\#\ of\ measurements\ required\ in\ joint\ reconstruction}{\#\ of\ measurements\ required\ in\ individual\ reconstruction}$

For simulation purpose we take total number of channels, $N_T = 1000$, out of which, $N_c = 30\%$ are overlapping, the sparsity, ($\frac{K_T}{N_T} = 10\%$), and $SOF = 0.5$ unless stated otherwise. Compressed measurements at different sampling rate are obtained and reconstructed. The results provided are the average of 1000 simulations. Both noisy and noiseless measurement schemes are simulated.

From Figs. 3 and 4, it is clearly observed that for the same number of compressed measurements, the joint reconstruction algorithm has better performance than the individual reconstruction. We see that the POD approaches 1 for joint reconstruction at sampling rate of 30% whereas it is at 44% for the individual reconstruction. This gain in measurements is the consequence of sparse overlapping elements and joint reconstruction. The EOR for joint reconstruction approaches to zero for the sampling rate of as low as 28% where as for that of individual reconstruction it occurs at 40%. We see that for same performance the joint reconstruction requires less number of samples. This reduces the data acquisition cost and the redundancies.

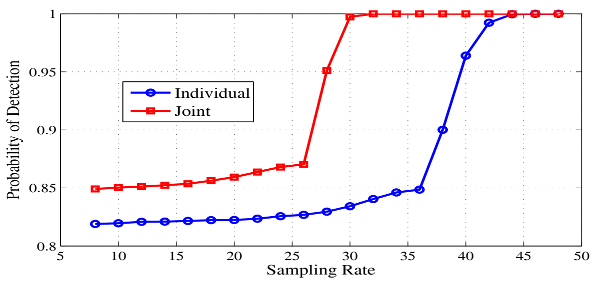

Fig. 3. POD using individual and joint reconstruction

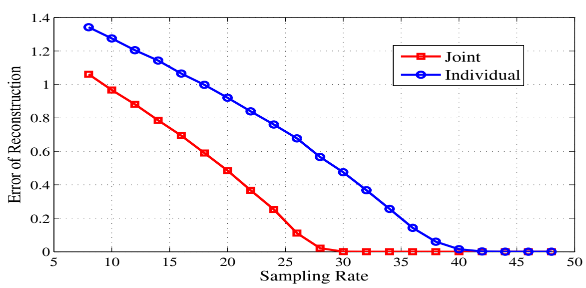

Fig. 4. EOR using individual and joint reconstruction

Figs. 5 and 6 are the performance measurement under noisy measurements.The performance under noisy measurements degrades both in terms of probability of detection and reconstruction error, however the joint reconstruction scheme still performs better than the individual reconstruction.

Fig. 7 shows the effect of the varying SOF on the measurement gain between the individual reconstruction and the joint reconstruction. It shows the measurement gain for $POD = 0.99$. We can clearly see that the measurement gain increases as the SOF increases. This implies that the number of measurements required in joint reconstruction for same performance decreases comparatively to individual reconstruction when there are more occupied channels in the overlapping region.

We also compare our performance with the iterative LASSO consensus scheme in [15]. In [15], the frequency overlapping scheme is illustrated using multihop cognitive network with

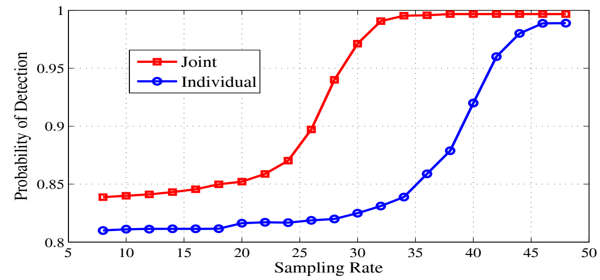

Fig. 5. POD using individual and joint reconstruction in noisy measurements

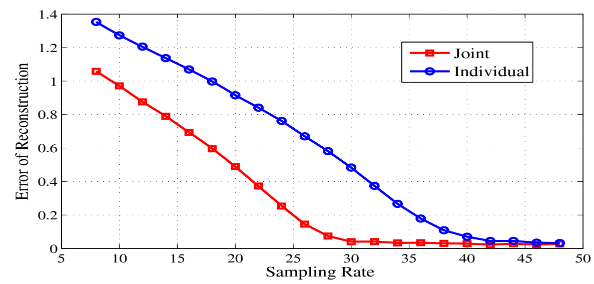

Fig. 6. EOR using individual and joint reconstruction in noisy measurements

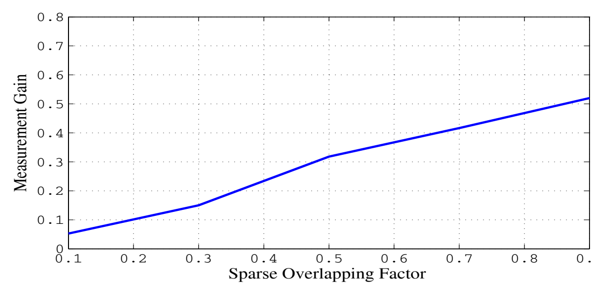

Fig. 7. Measurement gain for varying SOF when POD=0.99

some common bands and innovation bands between multiple hops in a network. Fig. 8 is the *Receiver operating characteristics (ROC)* comparison and Fig. 9 shows the comparison of reconstruction error. We clearly observe that the joint reconstruction scheme has better receiver operating characteristics where as the error of reconstruction is comparable to that of in iterative LASSO consensus.

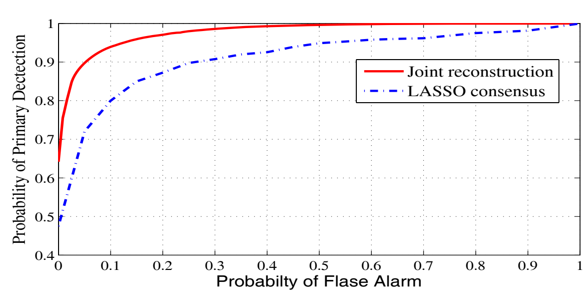

Fig. 8. ROC performance comparison, For SNR=-5dB, S.R=0.6 and Sparsity=40%

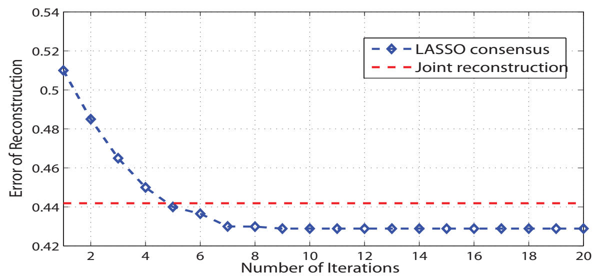

Fig. 9. EOR comparison, For SNR=-5dB, S.R=0.6 and Sparsity=40%

We also reconstruct the original time domain signal using individual and joint reconstruction methods in Figs. 10 and 11, respectively, at sampling rate of 32%. Comparing these figures, we observe that the signal reconstructed using the joint reconstruction matches more closely to the original signal.

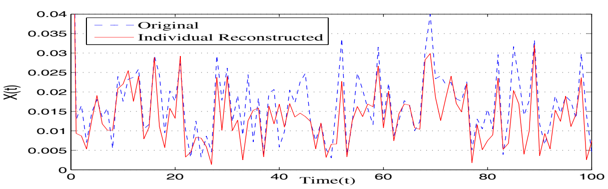

Fig. 10. Original time domain signal and reconstructed signal using individual reconstruction method

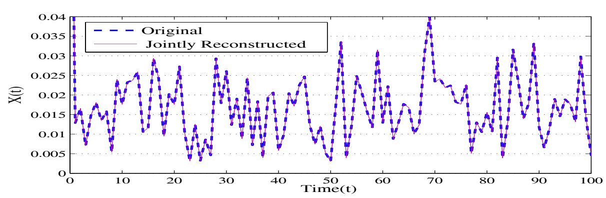

Fig. 11. Original time domain signal and reconstructed signal using joint reconstruction method

V. Conclusion

In this paper, we proposed a novel wide band spectrum sensing for cognitive radios in the frequency overlapping networks using distributed compressive sensing and joint reconstruction. The concept have been demonstrated through the theoretical explanation and have been validated using the simulation results. A distributed compressive sensing for cognitive radio network in the frequency overlapping system has been explored. Proposed joint reconstruction scheme for spectrum sensing exploits the joint sparsity in frequency overlapping networks and efficiently reduces the number of samples required. It is shown that the proposed scheme outperforms the individual reconstruction scheme and has better receiver operating characteristics compared to LASSO consensus algorithm. This is because the overlapping channels can be exploited to enhance the compressive decoding using joint reconstruction scheme.

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